

Theory of Optically-Driven Sideband Cooling for Atomic Collective Excitations and Its Generalization

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(Dated: June 4, 2009)

We explore how to cool atomic collective excitations in an optically-driven three-level atomic ensemble, which may be described by a model of coupled two harmonic oscillators (HOs) with a time-dependent coupling. Moreover, the coupled two-HO model is further generalized to address other cooling issues, where the lower-frequency HO can be cooled whenever the cooling process dominates over the heating one during the sideband transitions. Unusually, due to the absence of the heating process, the optimal cooling of our first cooling protocol for collective excitations in an atomic ensemble could break a usual sideband cooling limit for general coupled two-HO models.

PACS numbers: 03.65.-w, 37.10.De, 43.58.Wc

Introduction.- Recently, quantum information processing based on collective excitations in atomic ensembles has attracted more and more attentions. Photons are good carriers of quantum information due to their fast velocity and low leakage, while may not be easy to store. Naturally, it is desired to study atomic ensembles as potential quantum memory units of photons due to the long coherence time. Interestingly, the form-stable dark-state polariton (DSP) [1] associated with the propagation of quantum optical fields via electromagnetically induced transparency (EIT) [2], was proposed in a three-level Λ -type atomic ensemble. In the low excitations limit, DSP can be described as a hybrid bosonic mode [3]. By controlling the mixing angle between light and matter components of DSP, the optical pulse can be decelerated and “trapped” via mapping its shape and quantum state onto meta-stable collective-excitation state of matter. That means the quantum information storage [1, 3, 4] can be achieved in the atomic ensembles by adiabatically controlling the coupling.

As is known, collective excitations could also be used in quantum communication in atomic ensembles and linear optics. Since the work of Duan-Lukin-Cirac-Zoller [5], a number of protocols [6, 7, 8, 9] have been proposed to implement robust long-distance quantum communications, quantum repeaters, and quantum information storages based on atomic ensembles over long photonic lossy channels.

In a realistic atomic ensemble, a given collective-excitation mode may have a finite thermal population due to the interaction with the thermal bath at finite temperature. This means that it is necessary to cool the thermal excitations in quantum information processing based on atomic collective excitations. In this Letter, we consider a driven three-level atomic ensemble that can be modeled by coupled two harmonic resonators (HOs), and then elaborate how to cool the low-frequency collective-excitation mode near its ground state in this kind of systems.

On the other hand, various nano- (or submicron-) mechanical resonators have been investigated [10] extensively in recent years. To reveal the quantum effect in the nano-mechanical devices, various cooling schemes [11, 12, 13, 14, 15, 16] were proposed to drive them to reach the standard

quantum limit (SQL) [17]. A famous one among them is the optical radiation-pressure cooling scheme [12] attributed to the (resolved) sideband cooling [13, 14, 15, 16], which was previously well-developed to cool the spatial motion of the trapped ions [18] or the neutral atoms [19]. Notably, our cooling scheme for atomic ensembles is based on the sideband structure induced by the lower-frequency mode, which is time-dependently coupled with the higher-frequency mode to loss its energy. Moreover, we generalize the above coupled two-HO model to other two types of cooling model beyond the optical radiation-pressure cooling of mechanical resonator. In the generalized model, the lower-frequency HO can be cooled with a usual sideband cooling limit, whose cooling mechanism can also be employed to understand the cooling of collective excitations in the atomic ensembles. It is remarkable that our protocol of atomic ensemble breaks the limit of usual sideband cooling due to the absence of counter-rotating terms in such a coupled two-HO model.

Three-Level atomic ensemble modeled by two coupled oscillators.- Let us consider an ensemble of N identical three-level atoms as seen in Fig. 1(a). A strong classical driving light field is homogeneously coupled to each atomic transition from the metastable state $|b_0\rangle$ to the excited one $|a_0\rangle$. Then the Hamiltonian reads ($\hbar = 1$ hereafter)

$$H = \omega_a \sum_{i=1}^N \sigma_{a_0 a_0}^{(i)} + \omega_b \sum_{i=1}^N \sigma_{b_0 b_0}^{(i)} + (\Omega e^{i\omega_d t} \sum_{i=1}^N \sigma_{b_0 a_0}^{(i)} + h.c.), \quad (1)$$

where $\omega_{g,a,b}$ are the corresponding energies of the atomic states $|g_0\rangle$, $|a_0\rangle$ and $|b_0\rangle$ respectively, and the ground state energy $\omega_g = 0$. Ω is the coupling strength of the driving light field (with the carrier frequency ω_d), which can be assumed to be real.

Normally, a weak quantized probe light would couple to the transition $|g_0\rangle \rightarrow |a_0\rangle$. Thus, a so-called Λ -type three-level atomic ensemble configuration can be constructed associated with the well-known EIT and group-velocity slowdown phenomena. In such an ensemble, the DSP can also be obtained as the superposition of the optical mode and the atomic collective-excitation mode [1, 3, 4]. Based on the notations

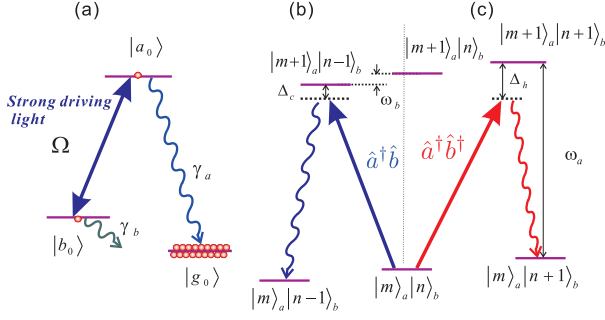


FIG. 1: (Color online) (a) Three-level atomic ensemble with most atoms staying in the ground states $|g_0\rangle$. The strong driving light couples to the transition from the meta-stable state $|b_0\rangle$ to the excited one $|a_0\rangle$ for each atom. The electric-dipole transition $|g_0\rangle \rightarrow |a_0\rangle$ is permitted, but $|g_0\rangle \rightarrow |b_0\rangle$ is forbidden. The wavy lines denote the decay processes with $\gamma_{a,b}$ the corresponding decay rates. (b) The cooling process ($|n\rangle_b \rightarrow |n-1\rangle_b$) and (c) the heating process ($|n\rangle_b \rightarrow |n+1\rangle_b$) for mode b starting from $|m\rangle_a|n\rangle_b$ in the sideband structure forming by splitting a-mode with the low-frequency b-mode. $\Delta_c (\equiv \omega_a - \omega_b - \omega_d)$ and $\Delta_h (\equiv \omega_a + \omega_b - \omega_d)$ are the detunings for the anti-Stokes (cooling) and Stokes (heating) transitions, respectively.

of EIT and DSP, the atomic ensemble can be a unit of quantum memory and be used to store the quantum information of, e.g., the photons. Here, we focus only on the cooling of the atomic collective excitations in the absence of the probe light field, also noting that extensive studies have been made in the framework of optically-pumping an individual atom into its internal lowest-energy ground state [20].

We now introduce the bosonic operators $\hat{a} = \sum_i \hat{\sigma}_{g_0 a_0}^{(i)} / \sqrt{N}$ and $\hat{b} = \sum_i \hat{\sigma}_{g_0 b_0}^{(i)} / \sqrt{N}$ for atomic collective excitations [3, 21], which satisfy $[\hat{a}, \hat{a}^\dagger] = 1$, $[\hat{b}, \hat{b}^\dagger] = 1$ and $[\hat{a}, \hat{b}^\dagger] = 0 = [\hat{a}, \hat{b}]$ in the limit of $N \rightarrow \infty$ with low excitations. Then, Hamiltonian (1) is modeled by the coupled two-HO model, and can be further rewritten in a time-independent form in the rotating framework as

$$H_I = \Delta \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \Omega (\hat{a}^\dagger \hat{b} + h.c.) \quad (2)$$

with the detuning $\Delta \equiv \omega_a - \omega_d$. In the derivation of the above Hamiltonian, we have used the rotating wave approximation (RWA) when $\{|\omega_{ab} - \omega_d|, |\Omega|\} \ll (\omega_{ab} + \omega_d)$ (where $\omega_{ab} \equiv \omega_a - \omega_b$), which is always fulfilled for most realistic atoms.

Sideband cooling for atomic collective excitations. - Generally, the atomic collective-excitation modes have non-vanishing mean thermal populations due to their couplings to the bath at finite temperature. In experiments, the frequency of the higher-frequency atomic collective-excitation, i.e., mode a , is of the order of $2\pi \times 10^{14}$ Hz, which implies that its mean thermal excitation number can be considered as zero even at room temperature. Usually, the atomic ground state $|g_0\rangle$ and meta-stable one $|b_0\rangle$ are selected as the atomic two hyperfine levels with the frequency difference being the order of $2\pi \times 10^9$ Hz. Although there is no optical dipole transition between $|b_0\rangle$ and $|g_0\rangle$ because of the electric dipole transition rule, the decay from $|b_0\rangle$ to $|g_0\rangle$ still

exists due to the atomic collision or some other cases, with a very low decay rate. Such a very-low decay rate means that the lower-energy mode b possesses a long coherence time, which is just a distinct advantage of using the atomic collective excitations as quantum memory units. However, in consideration of the high initial mean thermal population $\bar{n}_b = [\exp(\omega_b/k_B T) - 1]^{-1} \sim 10^4 \gg 1$ at room temperature $T \sim 300$ K (with k_B the Boltzmann constant), it is necessary to cool the atomic collective-excitation modes to their ground states before quantum information processing based on atomic collective excitations.

In the presence of noises, we may have the following Langevin equation from Hamiltonian (2)

$$\dot{\hat{C}} = -\Gamma_C \hat{C} + i\Omega \hat{C}' + \hat{F}_C(t), \quad (3)$$

where $C, C' = a, b$ ($C \neq C'$), $\Gamma_a = \gamma_a/2 + i\Delta$ and $\Gamma_b = \gamma_b/2 + i\omega_b$. The noise operators are described by the correlations $\langle \hat{F}_C^\dagger(t) \hat{F}_C(t') \rangle = \gamma_C \bar{n}_C \delta(t - t')$. Here, $\gamma_{a,b}$ are the decay rates of collective-excitation modes a and b , respectively (for simplicity, we adopt the same symbols as those of the atomic levels $|a_0\rangle$ and $|b_0\rangle$), and $\bar{n}_{a,b} = [\exp(\omega_{a,b}/k_B T) - 1]^{-1}$ are the corresponding initial thermal populations with T the initial temperature of the thermal bath. Although the above quantum Langevin equation has vanishing steady state solutions $\langle \hat{a} \rangle = \langle \hat{b} \rangle = 0$, the corresponding quantum rate equations for the excitation numbers $\hat{n}_C = \hat{C}^\dagger \hat{C}$ ($C = a, b$) read

$$\frac{d}{dt} \langle \hat{n}_C \rangle = \gamma_C (\bar{n}_C - \langle \hat{n}_C \rangle) - (i\Omega \langle \hat{\Sigma} \rangle + h.c.), \quad (4)$$

$$\frac{d}{dt} \langle \hat{\Sigma} \rangle = -\zeta \langle \hat{\Sigma} \rangle + ig(\langle \hat{n}_b \rangle - \langle \hat{n}_a \rangle), \quad (5)$$

where $\hat{\Sigma} = \hat{a}^\dagger \hat{b}$ and $\zeta = (\gamma_a + \gamma_b)/2 + i(\omega_b - \Delta)$. Here we have used the non-vanishing noise-based relations [22] $\langle \hat{F}_C^\dagger(t) \hat{C}(t) \rangle = \gamma_C \bar{n}_C/2$.

The steady state solutions of the quantum rate equations give the variation of final mean population $\bar{n}_b^f = \langle (\hat{b}^\dagger - \langle \hat{b}^\dagger \rangle)(\hat{b} - \langle \hat{b} \rangle) \rangle_{ss} \equiv \bar{n}_b - \xi(\bar{n}_b - \bar{n}_a)$ with

$$\xi = \frac{\Omega^2 \gamma_a (\gamma_a + \gamma_b)}{(\gamma_a + \gamma_b)^2 (\Omega^2 + \frac{\gamma_a \gamma_b}{4}) + \gamma_a \gamma_b (\Delta - \omega_b)^2}.$$

Then, from the Bose-Einstein distribution, the effective temperature T_{eff} of mode b is expressed as

$$T_{\text{eff}} = \frac{\omega_b}{k_B \ln(1/\bar{n}_b^f + 1)}. \quad (6)$$

For a simple case of $\Delta = \omega_b$ (namely, the driving light is exactly resonant to the atomic transition $|b_0\rangle \rightarrow |a_0\rangle$: $\omega_d = \omega_{ab}$), the nice cooling reaches with

$$\bar{n}_b^f = \frac{\gamma_b \bar{n}_b + \gamma_a \bar{n}_a}{\gamma_a + \gamma_b} \approx \frac{\gamma_b}{\gamma_a} \bar{n}_b + \bar{n}_a \quad (7)$$

in the strong driving strength limit $\Omega \gg \gamma_a, \gamma_b$. For a realistic atomic system, one has $\gamma_a \gg \gamma_b$ and $\bar{n}_b \gg \bar{n}_a$ ($\omega_a \gg \omega_b$).

Especially, when γ_b is sufficiently small such that $\gamma_b \bar{n}_b \ll \gamma_a \bar{n}_a$ [23], the final mean population reaches its limit: $\bar{n}_b^f \rightarrow \bar{n}_b^{\text{lim}} = \bar{n}_a$.

As mentioned above, the mean thermal population of mode a is usually tiny, which means that the atomic collective-excitation mode b can be cooled close to its ground state with the final thermal population $\bar{n}_b^f \rightarrow \bar{n}_a \ll 1$.

A physical explanation of the above results can resort to the sideband-cooling-like mechanism (see Fig. 1(b)). The Jaynes-Cummings (JC) term $(\hat{a}^\dagger \hat{b})$ causes the anti-Stokes transition from $|m\rangle_a |n\rangle_b$ to $|m+1\rangle_a |n-1\rangle_b$, which will decay fast to the state $|m\rangle_a |n-1\rangle_b$. Thus, such a process makes the lower-frequency oscillator b to lose one quantum and then results in its cooling. When the anti-Stokes transition is resonantly coupled, namely, $\Delta = \omega_b$, or $\Delta_c \equiv \omega_{ab} - \omega_d = 0$, the best cooling happens with the corresponding optimal final mean population (\bar{n}_b^f) given by the initial population \bar{n}_a of higher-frequency mode a . All in all, in order to reach the optimal cooling of lower-energy collective-excitation mode b , the following conditions should be satisfied: (i) strong enough pumping light $\Omega \gg \gamma_a, \gamma_b$; (ii) the resonantly driving condition: $\Delta_c \equiv \omega_{ab} - \omega_d = 0$; (iii) $\gamma_b \ll \gamma_a$ and $\bar{n}_a \ll \bar{n}_b$ (that is, $\omega_b \ll \omega_a$). It is notable that the above three conditions can be met for experimentally accessible parameters of realistic atomic systems [23].

It is seen clearly from the above analysis that the time-dependent coupling between two large-detuned HOs could cool down the lower-frequency one. This cooling model is different from the existing mechanical cooling scheme based on the optical radiation pressure [12, 13, 14, 15, 16], with an external laser-driving. Nevertheless, we will show below that these two cooling schemes may be generalized to a more universal model.

Generalized sideband cooling model of two coupled HOs. A naive cooling process could be realized when a hotter object contacts directly with a cold one. If there exists no external driving for two objects at the same initial temperature, it is obviously impossible that the temperature of any one can change via their direct interaction. But the situation changes dramatically when we add an additional time-dependent driving or manipulate the coupling between them to be time-dependent in largely-detuned two coupled HOs. This kind of setup leads to a more general sideband cooling framework.

Let us first consider two coupled HOs with large-detuned frequencies ($\omega_a \gg \omega_b$) as seen in Fig. 2(a). The free Hamiltonian reads $\hat{H}_0 = \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b}$. A time-dependent coupling is generally expressed as $\hat{V}_1(t) = g \cos(\omega_d t) F_1(\hat{a}^\dagger, \hat{a})(\hat{b}^\dagger + \hat{b})/2$, where \hat{a}^\dagger (\hat{a}) and \hat{b}^\dagger (\hat{b}) are the creation (annihilation) operators of the oscillators a and b with g the coupling coefficient between them and ω_d the modulating frequency. Here, $F_1(\hat{a}^\dagger, \hat{a})$ is a function of operators \hat{a}^\dagger and \hat{a} . For simplicity, in what follows we consider only the simplest case of $F_1(\hat{a}^\dagger, \hat{a}) = \hat{a}^\dagger + \hat{a}$, though a more general function (i.e., $F_1(\hat{a}^\dagger, \hat{a}) = \sum_n c_n \hat{a}^{\dagger n} (\hat{a}^\dagger + \hat{a}) \hat{a}^n$ with c_n dimensionless coefficients) would lead to a similar result. In the time-varying frame reference defined by $\hat{R}(t) = \exp(-i\omega_d \hat{a}^\dagger \hat{a} t)$, the ef-

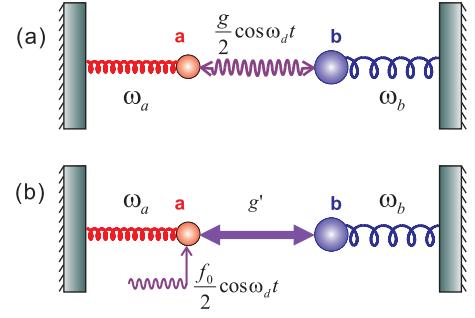


FIG. 2: (Color online) Coupled two HOs (a and b) model. b is the desired lower-frequency HO to be cooled. (a) The interaction between the HOs is time-dependent modulated ($\propto g \cos(\omega_d t)/2$); (b) The coupling strength of the interaction between two HOs is time-independent but there is an external time-dependent driving ($\propto f_0 \cos(\omega_d t)/2$) on the higher-frequency mode a .

fective Hamiltonian of the coupled system reads

$$\hat{H}_{\text{eff}} = \Delta \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + g(\hat{a}^\dagger + \hat{a})(\hat{b}^\dagger + \hat{b}), \quad (8)$$

where the high-oscillating terms have been neglected and the detuning $\Delta = \omega_a - \omega_d$ could be negative when $\omega_a < \omega_d$.

Next we consider another type of two-HO system (see Fig. 2 (b)): a general time-independent interaction is $\hat{V}_2 = g' F_2'(\hat{a}^\dagger, \hat{a}')(\hat{b}^\dagger + \hat{b}')$ with g' the coupling strength and $F_2'(\hat{a}^\dagger, \hat{a}')$ being Hermitian, and a periodically driving field on the higher-frequency HO reads $\hat{H}_d(t) = f_0 \cos(\omega_d t)(\hat{a}^\dagger + \hat{a}')/2$. In the time-varying frame reference defined by $\hat{R}'(t) = \exp(-i\omega_d \hat{a}'^\dagger \hat{a}' t)$, the total Hamiltonian reads $\hat{H} = \Delta_0 \hat{a}'^\dagger \hat{a}' + \omega_b \hat{b}^\dagger \hat{b}' + g' F_2'(\hat{a}'^\dagger, \hat{a}')(\hat{b}^\dagger + \hat{b}') + f_0(\hat{a}'^\dagger + \hat{a}')/2$ with $\Delta_0 = \omega_a - \omega_d$ after neglecting the high-oscillating terms, where $F_2'(\hat{a}'^\dagger, \hat{a}')$ keeps the time-independent terms in $F_2'(\hat{a}'^\dagger e^{i\omega_d t}, \hat{a}' e^{-i\omega_d t})$. Around some quasi-classical state $|Q\rangle$ such that $\langle Q | \hat{a}' | Q \rangle = \alpha$ and $\langle Q | \hat{b}' | Q \rangle = \beta$, the quantum dynamics is determined by an effective Hamiltonian $\hat{H}_{\text{eff}} = \hat{H}_{\text{eff}}(\hat{a}'^\dagger, \hat{b}'^\dagger, \hat{a}, \hat{b})$ with the displacement operators $\hat{a} = \hat{a}' - \alpha$ and $\hat{b} = \hat{b}' - \beta$ for quantum fluctuations. Then, when the displacements β and α take the equilibrium values $\beta = -F_2(\alpha, \alpha)/\omega_b$ and $\alpha = -[f_0 + 2\beta \partial_\alpha F_2(\alpha, y)|_{y=\alpha}]/\Delta_0$, the effective Hamiltonian \hat{H}_{eff} has the same form as that given in Eq. (8) with the parameters $\Delta = \Delta_0 + 2\beta [\partial^2 F_2(x, y)/\partial x \partial y]|_{x, y=\alpha}$ and $g = g' \partial_\alpha F_2(\alpha, y)|_{y=\alpha}$. Therefore, these types of coupled two-HO model should have the same cooling mechanism to cool the lower-frequency HO mode.

We wish to point out that the optical radiation-pressure cooling of mechanical resonator [13, 14, 15, 16], is just a special case of the second type with $F_2'(\hat{a}'^\dagger, \hat{a}') = g' \hat{a}'^\dagger \hat{a}'$. A similar linearization [13, 16] of the effective Hamiltonian as given in Eq. (8) was also mentioned in the optical radiation-pressure cooling of mechanical resonator. Here we present only the cooling limit (so-called sideband cooling limit) [24] of the general coupled two-HO model:

$$\bar{n}_b^f \rightarrow \bar{n}_b^{\text{lim, sid}} = \bar{n}_a + \frac{\gamma_a^2}{4\omega_b^2} \approx \frac{\gamma_a^2}{4\omega_b^2} \quad (9)$$

in the resolved sideband case $\gamma_a^2 \ll \omega_b^2$ when $\Delta = \sqrt{\omega_b^2 + \gamma_a^2} \approx \omega_b$. Here the usual relation $\bar{n}_a \ll \gamma_a^2/4\omega_b^2$ has been used.

Although the above Hamiltonian (8) describes only a simple coupled two-HO system, it can capture the essence of almost all sideband cooling schemes. We need to emphasize the necessity of the time-dependence of modulating coupling or external driving. It lies in a fact that, when $\omega_a \gg \omega_b$, there still exists the effective interaction for $|\Delta| \sim \omega_b$ (or $\omega_a \pm \omega_b \sim \omega_d$). It is the effective resonance $|\Delta| \sim \omega_b$ that results in the sideband transitions to cool down (or heat up) the oscillator b (see Fig. 1(b) and (c)): the JC term ($\hat{a}^\dagger \hat{b}$) (associated with the fast decay of mode a) denotes the cooling process of lower-frequency oscillator b ($|n\rangle_b \rightarrow |n-1\rangle_b$); on the contrary, the anti-JC term (that is, the anti-rotating term) ($\hat{a}^\dagger \hat{b}^\dagger$) denotes the heating process of mode b ($|n\rangle_b \rightarrow |n+1\rangle_b$). When the cooling process dominates (e.g., when $\Delta_c \sim 0$), the cooling of mode b happens with the optimal cooling subject to the usual sideband cooling limit ($\bar{n}_b^{\text{lim, sid}} \approx \gamma_a^2/4\omega_b^2$).

Comparing the cooling models described by the Hamiltonians (2) and (8), it is clear that the anti-JC terms ($\hat{a}^\dagger \hat{b}^\dagger + h.c.$) are absent in the former describing the atomic ensemble. Thus, due to absence of the heating process induced by the anti-JC term during the resolved sideband cooling, the optimal cooling of lower-frequency collective-excitation happens at the exact resonant ($\Delta_c = 0$) of (first) anti-Stokes transi-

tions and the corresponding cooling limit ($\bar{n}_b^{\text{lim}} = \bar{n}_a$) is certainly much less than that of the usual sideband cooling limit ($\bar{n}_b^{\text{lim, sid}} \approx \gamma_a^2/4\omega_b^2$).

Conclusion.- We have established a theory to cool atomic collective excitations in an optically-driven three-level atomic ensemble. Such a cooling protocol is quite useful and promising in quantum information processing based on atomic collective excitations, which breaks the usual sideband cooling limit. Moreover, motivated by the optical radiation-pressure cooling scheme of mechanical oscillator, we have also proposed two generalized cooling types of the coupled two-HO model: the first one possesses a time-dependent modulating coupling coefficient between the HOs without the external driving; while for the second one, an additional external time-dependent driving on the higher-frequency HO is involved, with the coupling coefficient between the HOs being time-independent. In fact, the second type is a generalized model of the optical radiation-pressure cooling of mechanical resonator. For both types, the lower-frequency HO can be cooled in the resolved sideband cooling case with the usual sideband cooling limit.

This work was supported by the RGC of Hong Kong under Grant No. HKU7051/06P, and partially supported by the NFRP of China under Grant Nos. 10874091 and 2006CB921205 and NSFC Grants through ITP, CAS.

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- [1] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000); Phys. Rev. A **65**, 022314 (2002).
 - [2] K.-J. Boller, A. Imamolu, and S. E. Harris, Phys. Rev. Lett. **66**, 2593 (1991); S. E. Harris, Phys. Today **50**, No. 7, 36 (1997); M. Xiao *et al.*, Phys. Rev. Lett. **74**, 666 (1995); H. Wu *et al.*, Phys. Rev. Lett. **100**, 173602 (2008).
 - [3] C. P. Sun, Y. Li, and X. F. Liu, Phys. Rev. Lett. **91**, 147903 (2003); Y. Li and C. P. Sun, Phys. Rev. A **69**, 051802(R) (2004).
 - [4] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
 - [5] L.-M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature **414**, 413 (2001).
 - [6] A. Kuzmich *et al.*, Nature **423**, 731 (2003); C. W. Chou *et al.*, Science **316**, 1316 (2007).
 - [7] B. Julsgaard *et al.*, Nature **432**, 482 (2004); M. D. Eisaman *et al.*, Nature **438**, 837 (2005).
 - [8] C. Liu *et al.*, Nature **409**, 490 (2001).
 - [9] B. Zhao *et al.*, Phys. Rev. Lett. **98**, 240502 (2007).
 - [10] A. N. Cleland and M. L. Roukes, Appl. Phys. Lett. **69**, 2653 (1996); X. M. H. Huang *et al.*, Nature **421**, 496 (2003).
 - [11] I. Wilson-Rae *et al.*, Phys. Rev. Lett. **92**, 075507 (2004); P. Zhang *et al.*, Phys. Rev. Lett. **95**, 097204 (2005); A. Naik *et al.*, Nature **443**, 193 (2006).
 - [12] C. H. Metzger and K. Karrai, Nature (London) **432**, 1002 (2004); S. Gigan, *et al.*, Nature (London) **444**, 67 (2006); O. Arcizet *et al.*, Nature (London) **444**, 71 (2006);
 - [13] I. Wilson-Rae *et al.*, Phys. Rev. Lett. **99**, 093901 (2007); T. J. Kippenberg and K. J. Vahala, Opt. Express **15**, 17172 (2007).
 - [14] F. Marquardt *et al.*, Phys. Rev. Lett. **99**, 093902 (2007); F. Marquardt *et al.*, J. Mod. Opt. **55**, 3329 (2008).
 - [15] C. Genes *et al.*, Phys. Rev. A **77**, 033804 (2008); Y. Li *et al.*, **78**, 134301 (2008); M. Grajcar *et al.*, Phys. Rev. B **78**, 035406 (2008).
 - [16] P. Rabl, C. Genes, K. Hammerer, and M. Aspelmeyer, arXiv: 0903.1637; S. Gröblacher, K. Hammerer, M. R. Vanner, M. Aspelmeyer, arXiv: 0903.5293.
 - [17] M. D. LaHaye, O. Buu, B. Camarota, and K. C. Schwab, Science **304**, 74 (2004).
 - [18] F. Diedrich *et al.*, Phys. Rev. Lett. **62**, 403 (1989); C. Monroe *et al.*, Phys. Rev. Lett. **75**, 4011 (1995).
 - [19] S. E. Hamann *et al.*, Phys. Rev. Lett. **80**, 4149 (1998).
 - [20] S. Chu, Science **253**, 861 (1991); C. S. Wood *et al.*, Science **275**, 1759 (1997).
 - [21] G. R. Jin *et al.*, Phys. Rev. B **68**, 134301 (2003).
 - [22] M. O. Scully and M. S. Zubairy, *Quantum Optics*, Chap. 9 (Cambridge University Press, New York, 1997).
 - [23] For typical alkali-(like)-metal Λ -type atoms, $\omega_a/2\pi \sim 10^{14}$ Hz, $\omega_b/2\pi \sim 10^9$ Hz, $\gamma_a/2\pi \sim 10^7$ Hz, $\gamma_b/2\pi \sim 10^{0-3}$ Hz (See Refs. [2, 8, 25] and references therein). The condition $\gamma_b \bar{n}_b \ll \gamma_a \bar{n}_a$ may be fulfilled for $\gamma_b/2\pi \sim 10^{0-1}$ Hz.
 - [24] For detailed calculations, one may refer to those in [13, 14, 15].
 - [25] A. Mosk *et al.*, Appl. Phys. B **73**, 791 (2001).